

Quantum superconductor-insulator transition: Implications of BKT-critical behavior

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We explore the implications of Berezinskii-Kosterlitz-Thouless (BKT) critical behavior on the two dimensional (2D) quantum superconductor-insulator (QSI) transition driven by the tuning parameter x . Concentrating on the sheet resistance $R(x, T)$ BKT behavior implies: an explicit quantum scaling function for $R(x, T)$ along the superconducting branch ending at the nonuniversal critical value $R_c = R(x_c)$; a BKT-transition line $T_c(x) \propto (x - x_c)^{z\bar{\nu}}$ where z is the dynamic and $\bar{\nu}$ the exponent of the zero temperature correlation length; independent estimates of $z\bar{\nu}$, z and $\bar{\nu}$ from the x dependence of the nonuniversal parameters entering the BKT expression for the sheet resistance. To illustrate the potential and the implications of this scenario we analyze data of Bollinger *et al.* taken on gate voltage tuned epitaxial films of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ that are one unit cell thick. The resulting estimates $z \simeq 2.35$ and $\bar{\nu} \simeq 0.63$ point to a 2D-QSI critical point where hyperscaling, the proportionality between $d/\lambda^2(0)$ and T_c , and the correspondence between quantum phase transitions in D and classical ones in (D+z) dimensions are violated and disorder is relevant.

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I. INTRODUCTION

A variety of different materials undergo a quantum superconductor-insulator (QSI) transition in the limit of two dimensions (2D) and zero temperature by variation of a tuning parameter including film thickness, disorder, applied magnetic field, and gate voltage.¹⁻⁴ A widespread observable to study this behavior is the temperature dependence of the sheet resistance $R(x, T)$ taken at various values of the tuning parameter x . The curves of $R(T)$ at different x resemble the flow to a nearly temperature independent separatrix between superconducting and insulating phase with sheet resistance $R_c = R(x = x_c, T \simeq 0)$. This behavior implies a crossing point of the isotherms $R(x)$ at different temperatures at x_c which is a characteristic feature of a quantum phase transition and in the present case of a QSI transition. Traditionally the interpretation of experimental data taken close to the 2D-QSI transition is based on the quantum scaling relation $R(x, T) = R_c(x) G(y)$ with $y = c(x - x_c)/T^{1/z\bar{\nu}}$.⁵⁻⁸ z is the dynamic, $\bar{\nu}$ the critical exponent of the zero temperature correlation length and c a nonuniversal coefficient of proportionality.¹⁻³ Given sheet resistance data fits to this scaling form yield estimates for the critical value of the tuning parameter x_c and the exponent product $z\bar{\nu}$, properties which are insufficient to distinguish different models, to fix the universality class to which the QSI transition belongs, or to clarify the relevance of disorder.

The nature of the 2D-QSI transition has been intensely debated.¹⁻⁴ The scenarios can be grouped into two classes, fermionic and bosonic. In the fermionic case the reduction of T_c and the magnitude of the order parameter is attributed to a combination of reduced density of states, enhanced Coulomb interaction and depairing due to an increase of the inelastic electron-electron scattering rate.^{9,10} The bosonic approach assumes that the fermionic degrees of freedom can be integrated out, the mean square of the order parameter does not vanish at

T_c , phase fluctuations dominate and the reduction of T_c is attributable to quantum fluctuations and in disordered systems to randomness in addition.^{6,7,11} This scenario is closely related to the suppression of ferroelectricity,¹² *e.g.* in SrTiO_3 .¹³

Here we adopt the bosonic scenario and concentrate on systems which undergo at finite temperature a normal state to superconductor transition with Berezinskii-Kosterlitz-Thouless (BKT) critical behavior,^{14,15} originally derived for the 2D xy-model with a two component order parameter and short range interactions. The occurrence of BKT criticality in 2D superconductors also implies that the mean square of the order parameter does not vanish at T_c and with that there are, in analogy to ^4He , condensed pairs (bosons) below and uncondensed ones above T_c . In ^4He and superconductors the order parameter is a complex scalar corresponding to the components in the xy-model. Supposing that in superconductors the interaction of Cooper pairs is short ranged and their effective charge is sufficiently small the critical properties at finite temperature are then those of the 3D-xy (bulk) and 2D-xy (thin films) models,^{16,17} reminiscent to the lambda transition in bulk ^4He ¹⁸ and the BKT-transition in thin ^4He films.¹⁹⁻²² In this context it is important to recognize that the existence of the BKT-transition (vortex-antivortex dissociation instability) in ^4He films is intimately connected with the fact that the interaction energy between vortex pairs depends logarithmic on the separation between them. As shown by Pearl,²³ vortex pairs in thin superconducting films (charged superfluid) have a logarithmic interaction energy out to the characteristic length $\lambda_{2D} = \lambda^2/d$, beyond which the interaction energy falls off as $1/R$. Here λ is the magnetic penetration depth and d the film thickness. As λ_{2D} increases by approaching T_c the diamagnetism of the superconductor becomes less important and the vortices in a clean and thin superconducting film become progressively like those in ^4He films.^{24,25}

The occurrence of a 2D-QSI transition implies

a line $T_c(x)$ of BKT transition temperatures ending at the quantum critical point at $x = x_c$ where $T_c(x = x_c) = 0$. It separates the superconducting from the insulating ground state. The BKT transition is rather special because the correlation length diverges above T_c as $\xi(x, T) = \xi_0(x) \exp\left(\left(b_R(x)/2T_c^{1/2}(x)\right)(T/T_c(x) - 1)^{-1/2}\right)$ and the sheet resistance tends to zero according to $R(x, T) = R_0(x) \exp\left(-\left(b_R(x)/T_c^{1/2}(x)\right)(T/T_c(x) - 1)^{-1/2}\right)$.²⁴⁻²⁶ Approaching the 2D-QSI transition quantum phase fluctuations renormalize $R_0(x)$, $b_R(x)$, and $T_c(x)$. Indeed the BKT-transition line approaches the 2D-QSI transition as $T_c = (c(x - x_c)/y_c)^{z/\bar{\nu}}$ because the quantum scaling form $G(y)$ exhibits at the universal value y_c of the scaling argument a finite temperature singularity.^{6,8,16} Noting that the amplitude of the BKT correlation length $\xi_0(x)$ should match the divergence of the quantum counterpart, $\xi(T = 0) \propto (x - x_c)^{-\bar{\nu}} \propto T_c^{-1/z}$ the exponents z and $\bar{\nu}$ should emerge from the amplitude $R_0(x)$ in terms of $R_0(x) - R_c \propto \xi_0^{-2}(x) \propto \xi^{-2}(T = 0) \propto (x - x_c)^{2\bar{\nu}} \propto T_c^{2/z}$.²⁶ Accordingly, the nonuniversal functions $T_c(x)$ and $R_0(x)$ entering the BKT expression for the sheet resistance exhibit close to the QSI transition quantum critical properties disclosing the quantum critical exponents $z\bar{\nu}$, z and $\bar{\nu}$. The exponents z and $\bar{\nu}$ are characteristic properties of the universality class to which the QSI transition belongs. In addition, given their values the relevance of disorder and the equivalence between quantum phase transitions in systems with D spatial dimensions and the ones of classical phase transitions in $(D + z)$ dimensions can be checked.^{5,6,16} The occurrence of a BKT transition line also implies: a nonuniversal critical sheet resistance $R_c = R_0(x_c)$ because $R_0(x)$ is nonuniversal; an explicit form of the superconductor branch of the quantum scaling function $G(c(x - x_c)/T^{1/z\bar{\nu}})$.

Even though BKT critical behavior is not affected by short-range correlated and uncorrelated disorder^{27,28} the observation of this behavior requires sufficiently homogeneous films and a tuning parameter which does not affect the disorder. Noting that sample inhomogeneity and vortex pinning are relevant in thickness and perpendicular magnetic field tuned transitions, electrostatic tuning of the 2D-QSI transition using the electric field effect appears to be more promising.^{26,29-32} Indeed electrostatic tuning is not expected to alter physical or chemical disorder, but changes the mobile carrier density.

In Sec. II we sketch the theoretical background. To illustrate the potential and the implications of finite temperature BKT criticality on the 2D-QSI transition we analyze in Sec. II the sheet resistance data of Bollinger *et al.*³⁰ taken on gate voltage tuned epitaxial films of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ that are one unit cell thick. Commenting on the difficulties in observing the BKT features in the magnetic penetration depth we consider the data of Bert *et al.*³² taken on the superconducting $\text{LaAlO}_3/\text{SrTiO}_3$

interface.

II. THEORETICAL BACKGROUND

Continuous quantum-phase transitions (QPT) are transitions at zero temperature in which the ground state of a system is changed by varying a parameter of the Hamiltonian.^{5,16,17} The quantum superconductor-insulator transitions (QSI) in two-dimensional (2D) systems tuned by disorder, film thickness, magnetic field or with the electrostatic field effect are believed to be such transitions.^{1-3,16,17,26,30} Traditionally the interpretation of experimental data taken close to the 2D-QSI transition is based on the quantum scaling relation,⁵⁻⁸

$$\frac{R(x, T)}{R_c} = G(y), y = \frac{c|x - x_c|}{T^{1/z\bar{\nu}}}. \quad (1)$$

where R is the resistance per square, R_c the limiting ($x \rightarrow x_c$ and $T \rightarrow 0$) resistance. x denotes the tuning parameter and c a nonuniversal coefficient of proportionality. $G(y)$ is a universal scaling function of its argument such that $G(y = 0) = 1$. In addition, z is the dynamic and $\bar{\nu}$ the critical exponent of the correlation length, supposed to diverge as

$$\xi(T = 0) = \bar{\xi}_0 |x - x_c|^{-\bar{\nu}}. \quad (2)$$

The critical sheet resistance R_c separating the superconducting and insulating ground states is determined from the isothermal sheet resistance at the crossing point in $R(T)$ vs. tuning parameter x at x_c . The existence of such a crossing point, remaining temperature independent in the zero temperature limit, is the signature of a QPT. The data for $R(x, T)$ plotted vs. $|x - x_c|/T^{1/\bar{\nu}z}$ should then collapse onto two branches joining at R_c . The lower branch stems from the superconducting ($x - x_c > 0$) and the upper one from the insulating phase ($x - x_c < 0$). This scaling form follows by noting that the divergence of the zero temperature correlation length, $\xi(T = 0) = \xi_0 |\delta|^{-\bar{\nu}}$, is at finite temperature limited by the length $L_T \propto T^{-1/z}$.⁵ Thus $G(y)$ is a finite-size scaling function because $y \propto [L_T/\xi(T = 0)]^{1/\bar{\nu}} \propto |x - x_c|/T^{1/\bar{\nu}z}$. Supposing that there is a line of finite temperature phase transitions $T_c(x)$ ending at the quantum critical point $T_c(x = x_c) = 0$ the quantum scaling form (1) exhibits at the universal value y_c of the scaling argument a finite temperature singularity.^{6,8,16} The phase transition line is then fixed by

$$T_c = \left(\frac{c|x - x_c|}{y_c}\right)^{z\bar{\nu}}. \quad (3)$$

Otherwise one expects that sufficiently homogeneous 2D superconductors exhibit at the superconductor to normal state transition BKT critical behavior.^{14,15,26} Note that there is the Harris criterion,²⁷ stating that short-range correlated and uncorrelated disorder is irrelevant at the

unperturbed critical point, provided that $\nu > 2/D$, where D is the dimensionality of the system and ν the critical exponent of the finite-temperature correlation length. With $D = 2$ and $\nu = \infty$, appropriate for the BKT transition,^{14,15} this disorder should be irrelevant. Given a BKT superconductor to normal state transition the sheet resistance scales for $T \gtrsim T_c(x) \geq 0$ as²⁴⁻²⁶

$$\frac{R(x, T)}{R_0(x)} = \exp\left(-\frac{b_R(x)}{T_c^{1/2}(x) (T/T_c(x) - 1)^{1/2}}\right), \quad (4)$$

allowing to probe the characteristic BKT correlation length^{14,15,25,26}

$$\frac{\xi(x, T)}{\xi_0(x)} = \exp\left(\frac{b_R(x)}{2T_c^{1/2}(x) (T/T_c(x) - 1)^{1/2}}\right). \quad (5)$$

While $R_0(x)$, $b_R(x)$, and $T_c(x)$ depend on the tuning parameter and are subject to quantum fluctuations the characteristic BKT form of the correlation length and with that of the sheet resistance applies for any $T \gtrsim T_c(x) \geq 0$. Through standard arguments (see, e.g., Ref.³³) quantum mechanics does not modify universal finite temperature properties. $b_R(x)$ is given by^{15,26}

$$b_R(x) = \tilde{b}_R T_c^{-1/2}, \quad (6)$$

with $\tilde{b}_R = 4\pi/b$. The parameter b is expected to remain constant in the low T_c regime.²⁶ It is related to the vortex core energy E_c in terms of $b = f(E_c/k_B T_c)$ ³⁴ and controls below the Nelson-Kosterlitz jump the temperature dependence of the magnetic penetration depth in terms of $(\lambda(T_c)/\lambda(T))^2 = 1 + (b/4)(T/T_c - 1)^{1/2}$.³⁵ The amplitude of the BKT correlation length $\xi_0(x)$ is proportional to the vortex core radius¹⁹ known to increase with reduced T_c .^{20,22} Indeed, the zero temperature correlation length $\xi(T=0)$ diverges as $\xi(T=0) \propto (x - x_c)^{-\bar{\nu}}$ (Eq. (2) and combined with $T_c \propto (x - x_c)^{z\bar{\nu}}$ (Eq. (3)) we obtain $\xi(T=0) \propto (x - x_c)^{-\bar{\nu}} \propto T_c^{-1/z}$. Noting that $R_0(x)$ approaches R_c from above the scaling relation

$$R_0(x) - R_c \propto \xi^{-2}(T=0) \propto (x - x_c)^{2\bar{\nu}} \propto T_c^{2/z}, \quad (7)$$

should apply,²⁶ making the determination of the exponents $\bar{\nu}$ and z possible. Consistency requires that the resulting $z\bar{\nu}$ agrees with the estimate obtained from the critical BKT line $T_c(x) \propto (x - x_c)^{z\bar{\nu}}$. Other implications concern the universality class of the 2D-QSI transition and the relevance of disorder. Given estimates for $\bar{\nu}$ and z the equivalence between quantum phase transitions in clean systems with D spatial dimensions and the ones of classical phase transitions in $(D+z)$ dimensions can be checked. The fate of a clean critical point under the influence of disorder is controlled by the Harris criterion.^{27,28} If the zero temperature correlation length critical exponent fulfills the inequality $\bar{\nu} \geq 2/D$ the disorder does not affect the critical behavior. If the Harris criterion is violated, $\bar{\nu} < 2/D$, the generic result is a new critical point

with conventional power law scaling but new exponents which fulfill the Harris criterion. Another option is that the disorder destroys the QSI transition.

Finally, given a BKT transition line expression (4) the sheet resistance given by Eq. (5) transforms with Eq. (6) and the scaling variable $y = c|x - x_c|/T^{1/z\bar{\nu}}$ to

$$\ln\left(\frac{R(x, T)}{R_0(x)}\right) = -\tilde{b}_R \left(\left(\frac{y_c}{y}\right)^{z\bar{\nu}} - 1\right)^{-1/2}, \quad (8)$$

valid for $y \leq y_c$. Close to quantum criticality where $R_0(x) \simeq R_c$ reduces to

$$\begin{aligned} \ln\left(\frac{R(x, T)}{R_c}\right) &= -\tilde{b}_R \left(\left(\frac{y_c}{y}\right)^{z\bar{\nu}} - 1\right)^{-1/2} \\ &= \ln(G(y)). \end{aligned} \quad (9)$$

These relations are explicit forms of the quantum scaling function $G(y)$ applicable to the superconductor branch. They reveal that the critical sheet resistance $R_0(x \rightarrow x_c) = R_c$ is the endpoint of a nonuniversal function and accordingly nonuniversal. Another implication concerns the universality class of the 2D-QSI transition. Supposing that the equivalence between quantum phase transitions in clean systems with D spatial dimensions and the ones of classical phase transitions in $(D+z)$ dimensions applies, the 2D-QSI transition at the endpoint of a BKT line $T_c(x)$ should belong to the finite temperature $(2+z) - xy$ universality class. xy denotes an order parameter with two components, including the complex scalar, $\Psi = |\Psi| \exp(i\varphi)$, of a superconductor.^{5,16}

The BKT - theory of thermally-excited vortex-antivortex pairs also predicts a super-to-normal state phase transition marked by the Nelson-Kosterlitz jump, a discontinuous drop in superfluid density from³⁵,

$$\frac{d}{\lambda^2(T_c^-)} = \frac{32\pi^2 k_B T_c}{\Phi_0^2} \simeq 1.02 T_c, \quad (10)$$

to zero. The numerical relationship applies for $d/\lambda^2(T_c^-)$ in cm^{-1} and T_c in K. d denotes the thickness of the 2D system, λ the in-plane magnetic penetration depth, and $\Phi_0 = hc/2e$. In addition there is the prediction that $d/\lambda^2(T=0)$, a measure of the phase stiffness, scales near the endpoint of the BKT transition line as^{6,8,16,17}

$$\frac{d}{\lambda^2(T=0)} = \frac{16\pi^3 k_B T_c}{\Phi_0^2} Q_2 \simeq 1.6 Q_2 T_c, \quad (11)$$

provided that $D+z$ is below the upper critical dimension D_u where hyperscaling holds. Since $D_u = 4$ in the D -xy-model the validity of Eq. (11) is in $D = 2$ restricted to $z < 2$. Relation (11) is the quantum counterpart of the Nelson-Kosterlitz relation (8). Q_2 is a dimensionless critical amplitude with the lower bound^{16,17}

$$Q_2 \geq 2/\pi, \quad (12)$$

dictated by the characteristic temperature dependence of $d/\lambda^2(T)$ below the Nelson-Kosterlitz jump (Eq. (10)).^{16,17} Combining Eqs. (10) and (11) we obtain

$$\left(\frac{\lambda(T=0)}{\lambda(T_c)}\right)^2 = \frac{\rho_s(T=0)}{\rho_s(T_c)} = \frac{\pi}{2}Q_2, \quad (13)$$

where ρ_s is the superfluid density. The superfluid transition temperature T_c as a function of the superfluid density $\rho_s(T=0)$ has been measured in ^4He films for transition temperatures ranging from 0.3 to 1 K by Crowell *et al.*²¹ They studied ^4He films adsorbed in two porous glasses, aerogel and Vycor, using high-precision torsional oscillator and dc calorimetry techniques. The investigation focused on the onset of superfluidity at low temperatures as the ^4He coverage is increased. Their data yields $\rho_s(T=0) \simeq 15.3T_c$ with ρ_s in $\mu\text{moles}/\text{m}^2$ and T_c in K. Combined with the BKT transition line $\rho_s(T_c=0) = 8.73T_c$ we obtain $Q_2 \simeq 1.12$.

III. COMPARISON WITH EXPERIMENT

To illustrate the potential and the implications of the outlined BKT scenario we analyze next the data of Bollinger *et al.*³⁰ taken on epitaxial films of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ that are one unit cell thick. Very large electric fields and the associated changes in surface carrier density enabled shifts in the midpoint transition temperature T_c by up to 30 K. Hundreds of resistance versus temperature and carrier density curves were recorded and shown to collapse onto a single function, as the quantum scaling form (Eq. (1)) for a 2D-QSI transition predicts. The observed critical resistance is close to the quantum resistance for pairs, $R_Q = h/4e^2 \simeq 6.45$ k Ω . Our starting point is the temperature dependence of the sheet resistance taken at various gate voltages V_g where a BKT transition is expected to occur. As an example we depicted in Fig. 1a the data for $V_g = -1.5$ V. The observation of BKT-behavior requires that the data extend considerably below the mean-field transition temperature T_{c0} . We estimated it with the aid of the Aslamov-Larkin (AL) expression³⁶ for the conductivity, $\sigma(T, V_g) = \sigma_n(V_g) + \sigma_0/(T/T_{c0} - 1)$, with $\sigma_0 = \pi e^2/8h \simeq 1.52 \times 10^{-5} \Omega^{-1}$, where Gaussian fluctuations are taken into account and T_{c0} is the mean-field transition temperature. The resulting temperature dependence is included in Fig. 1a. It clearly reveals that the data extend considerably below $T_{c0} \simeq 12.5$ K. To establish and characterize BKT behavior below T_{c0} we invoke Eq. (4) and

$$\left(\frac{d \ln(R)}{dT}\right)^{-2/3} = \left(\frac{2}{b_R(T_c)}\right)^{2/3} (T - T_c(x)). \quad (14)$$

As indicated in Fig. 1b this relation is used to fix $b_R(T_c)$ and $T_c(x)$ while $R_0(x)$ is estimated by adjusting Eq. (4) with given $b_R(T_c)$ and $T_c(x)$ to the sheet resistance data.

Comparing the data with the respective lines we observe that the BKT regime is attained and that the BKT T_c is almost an order of magnitude lower than the mean-field counterpart. This uncovers a BKT transition from uncondensed to condensed Cooper pairs driven by strong phase fluctuations. On the other hand it is important to recognize that agreement with BKT-criticality is established in a temperature window only. Its upper bound reflects the crossover from BKT- to AL-behavior while the lower bound stems from the rounded BKT-transition. Precursors of this phenomenon are clearly visible in Fig. 1 around 7 K. Here the correlation length is prevented to grow beyond a limiting length L , *i.e.* the linear extent of the homogeneous domains. As a result, a finite size effect and with that a rounded transition occurs.²⁶ Because the BKT correlation length does not exhibit the usual power law divergence of the correlation length as T_c is approached, it is particularly susceptible to such finite-size effect.

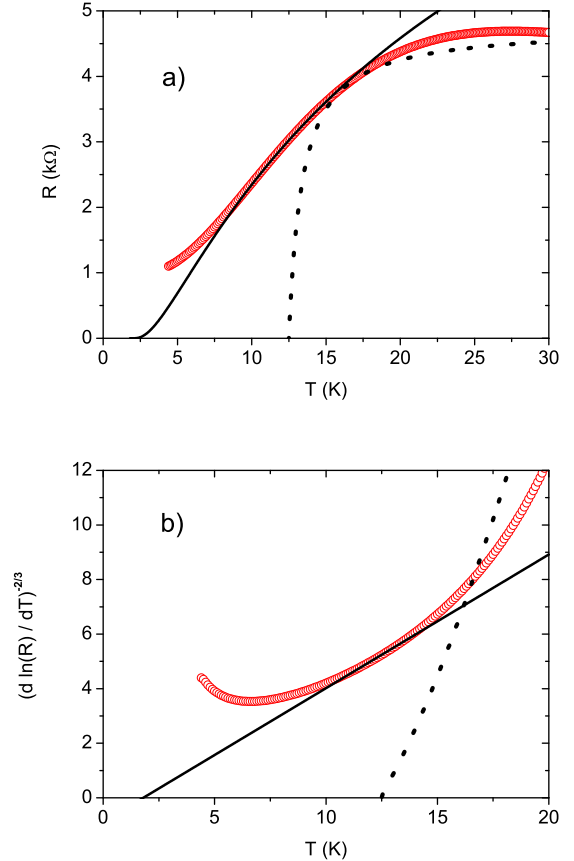


FIG. 1: (color online) a) Sheet resistance $R(T)$ for $V_g = -1.5$ V taken from Bollinger *et al.*³⁰ The dotted curve is a fit to the Aslamov-Larkin expression yielding the $T_{c0} \simeq 12.5$ K. The solid line is the BKT behavior Eq. (4) with $R_0 = 18$ k Ω and the T_c, b_R values derived in Fig. 1b. b) $(d \ln(R)/dT)^{-2/3}$ vs. T derived from the data shown in Fig. 1a. The dotted line indicates the AL-behavior and the solid one Eq. (13) with $T_c = 1.8$ K and $(2/b_R(T_c))^{2/3} = 0.49$ (K $^{-1/3}$)

To explore the quantum critical behavior latent in $T_c(V_g)$, $R_0(V_g)$ and $b_R(T_c)$ we performed this analysis of the temperature dependence of the sheet resistance for additional gate voltages. To fix the critical gate voltage V_{gc} we used the empirical gate voltage dependence of the number of mobile holes x per one formula unit of Bollinger *et al.*³⁰ yielding $x(V_g) = x_c + 0.012(V_{gc} - V_g)$ down to $V_g = -2$ V with $x_c = 0.0605$ and $V_{gc} \simeq -0.7$ V. The results $T_c(V_g)$, $R_0(V_g)$ are shown in Fig. 2a and Fig. 2b while $b_R(T_c)$ is depicted in Fig. 3. As can be seen in Fig. 2a the BKT transition line differs substantially from the so called superconducting dome behavior observed in bulk cuprate superconductors. It is approximately given by $T_c(x)/T_{c\max} = 1 - 82.6(x - x_m)^2$ where $T_{c\max}(x_m = 0.16)$ is the maximum T_c .³⁷ Close to the QSI transition where even bulk cuprate superconductors become essentially 2D¹⁷ it reduces to $T_c(x)/T_{c\max} \simeq 10.3(x - x_c)$ with $x_c \simeq 0.049$ and suggests $z\bar{\nu} \simeq 1$. In any case it differs substantially from the BKT line shown in Fig. 2a yielding the estimate

$$z\bar{\nu} \simeq 1.46, \quad (15)$$

in agreement with the value $z\bar{\nu} \simeq 1.5$ derived by Bollinger *et al.*³⁰ using the quantum scaling approach. In contrast to this, from the nonuniversal parameters $R_0(V_g)$ and $R_0(T_c)$ shown in Fig. 2b we derive with Eq. (7) in addition

$$\bar{\nu} \simeq 0.63, z \simeq 2.35, z\bar{\nu} = 1.48. \quad (16)$$

As these exponents satisfy the inequality $(2 + z)\bar{\nu} \geq 2$ we use the correct scaling argument, because $\delta = (\mu - \mu_c) \propto (x - x_c) \propto (V_{gc} - V_g)$ where μ denotes the chemical potential.⁶ The agreement between these $z\bar{\nu}$ values confirms the applicability of the scaling relation (7). Similarly, the nonuniversal parameter $b_R(x)$ exhibits according to Fig. 3 the expected T_c dependence (Eq. (6)). Noting that $D + z \simeq 2 + z \simeq 4.35$ exceeds the upper critical dimension $D_u = 4$ the critical exponent of the zero temperature correlation length $\bar{\nu}$ should adopt its mean-field value $\bar{\nu} = 1/2$. However the fate of this clean critical point under the influence of disorder is controlled by the Harris criterion.^{27,28} If the inequality $\bar{\nu} \geq 2/D$ is fulfilled, the disorder does not affect the critical behavior. If the Harris criterion is violated ($\bar{\nu} < 2/D$), the generic result is a new critical point with conventional power law scaling but new exponents which fulfill $\bar{\nu} < 2/D$. Since $\bar{\nu} = 1/2$ violates this inequality disorder is relevant and drives the system from the mean-field to an other critical point with different critical exponents as our estimate $\bar{\nu} \simeq 0.63$, fulfilling $\bar{\nu} < 2/D = 1$, uncovers. The resulting 2D-QSI transition with $\bar{\nu} \simeq 0.63$ and $z \simeq 2.35$ violates then the equivalence between quantum phase transitions in systems with D spatial dimensions and the ones of classical phase transitions in $(D + z)$ dimensions. In addition the proportionality between $d/\lambda^2(0)$ and T_c (Eq. (11)) valid below the upper critical dimension $D_u = 4$ does no longer hold because $(D + z) \simeq 4.35$ is above

$D_u = 4$. In fact magnetic penetration depth measurements taken on underdoped high quality $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ single crystals revealed $T_c \propto (d/\lambda^2(0))^{0.61}$.³⁸ Another option is that the disorder destroys the QSI transition. Given the evidence for a rounded BKT transition (see Fig. 1) and the missing data close to the QSI transition (see Fig. 4) further studies are required to elucidate this option.

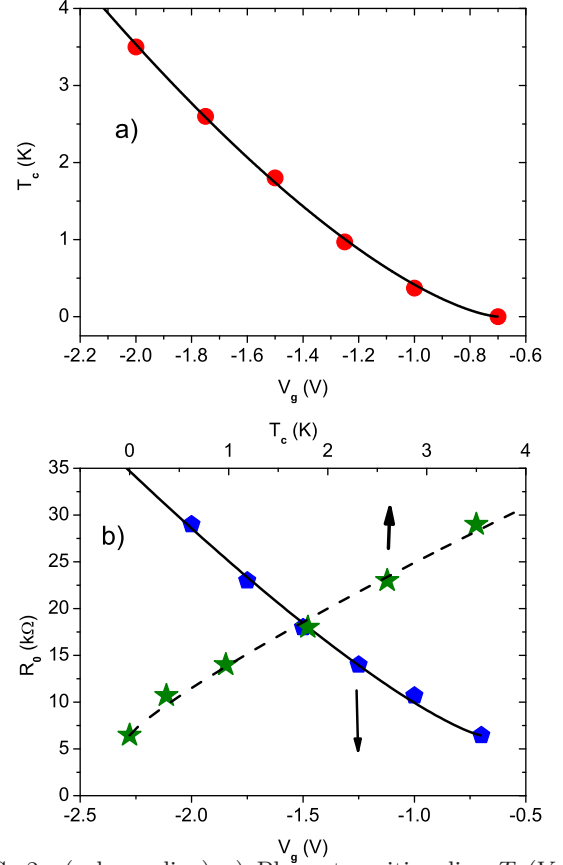


FIG. 2: (color online) a) Phase transition line $T_c(V_g)$ derived from the sheet resistance data of Bollinger *et al.*³⁰ using Eq. (14). The solid line is a least square fit yielding $T_c = 2.41(V_{gc} - V_g)^{z\bar{\nu}}$ (K) and $z\bar{\nu} = 1.46$ with $V_{gc} = -0.7$ V. b) $R_0(V_g)$ derived from the sheet resistance data of Bollinger *et al.*³⁰ using Eq. (4) with given $T_c(V_g)$ and $b_R(T_c)$; $R_0(T_c)$ derived from $R_0(V_g)$ using $T_c(V_g)$. The solid line is $R_0(V_g) = R_c + (V_{gc} - V_g)^{2\bar{\nu}}$ (kΩ) with $R_c = 6.45$ kΩ, $V_{gc} = -0.7$ V and $\bar{\nu} = 0.63$. The dashed line is $R_0(T_c) = R_c + 7.52T_c^{2/z}$ (kΩ) with $z = 2.35$.

To complete the analysis of the data of Bollinger *et al.*³⁰ we depicted in Fig. 4 the plot $R(V_g, T)/R_0(V_g)$ vs. $(T_c(V_g)/T)^{2/3}$ corresponding to the quantum scaling function $G(y)$ (Eq. (8)) in terms of $y/y_c = (T_c(V_g)/T)^{2/3}$. Apparently, the data does not fall completely on the BKT curve indicated by the dashed line. It corresponds to the superconductor branch of the quantum scaling function. Instead we observe a flow to and away from the universal characteristics. As T_c/T de-

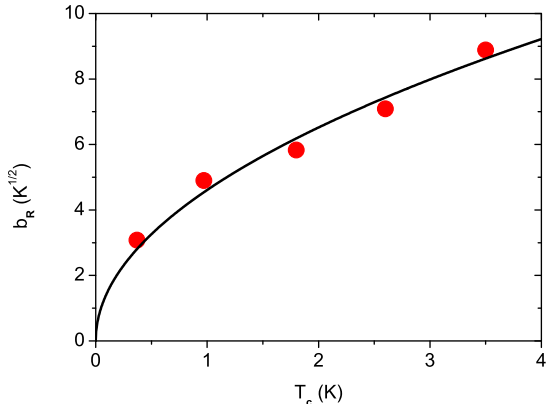


FIG. 3: (color online) Estimates for $b_R(T_c)$ derived from the sheet resistance data from Bollinger *et al.*³⁰ using Eq. (13). The solid line is $b_R(T_c) = \tilde{b}_R T_c^{1/2}$ ($\text{K}^{1/2}$) with $\tilde{b}_R = 4.61$ in agreement with Eq. (6).

creases for fixed T_c the crossover to AL-behavior sets in, while the rounding of the transition leads with increasing T_c/T to a flow away from criticality. The important lesson then is that the quality of the data collapse on a single curve heavily depends on the temperature range of the data entering the plot. Another striking feature is the extended scaling regime. Within the BKT scenario it simply follows from the fact that the scaling form (4) applies along the BKT transition line irrespective of the distance from the QSI transition. In the quantum scaling approach this property remains hidden and merely suggests an extended quantum critical regime. The excellent quality of the piecewise data collapse also reveals that the observance of the substantial variation of $R_0(V_g)$ (see Fig. 2b) is essential, while in the quantum scaling approach it is fixed by the critical sheet resistance. To demonstrate these features even more compelling we depicted in Fig. 5a R/R_c vs. $(V_{gc} - V_g)/T^{2/3}$. Apparently the data do not fall even piecewise on a single curve. After all this is not surprising because the quantum scaling form (1) holds close to the critical sheet resistance only and $R_0(V_g)$ varies substantially in the gate voltage regime considered here (see Fig. 2b). As shown in Fig. 5b and Fig. 6 this behavior allows to estimate $R_0(V_g)$ from R/R_c vs. $(V_{gc} - V_g)/T^{2/3}$ by rescaling R_c in terms of $b(V_g) R_c$ with $b(V_g = -1\text{V}) = 1$ to achieve piecewise a collapse of the data. The resulting $b(V_g)$ is shown in Fig. 6 and agrees well with $R_0(V_g)/R_0(V_g = -1\text{V})$ derived from the BKT behavior of the sheet resistance (see Fig. 2b).

A BKT line with a QSI transition at its endpoint was also explored rather detailed at the interface between the insulating oxides LaAlO_3 and SrTiO_3 exhibiting a superconducting 2D electron system that can be modulated by a gate voltage.^{26,32,39,40} BKT behavior and with that a 2D electron system was established as follows: The the current-voltage characteristics³⁹ revealed

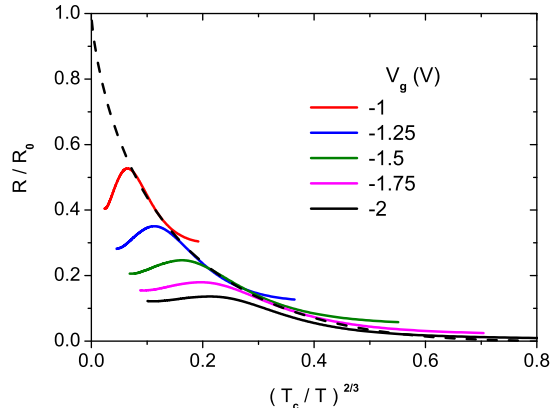


FIG. 4: (color online) $R(V_g, T)/R_0(V_g)$ vs. $(T_c(V_g)/T)^{2/3} = y/y_c$ corresponding to the BKT quantum scaling function $G(y)$ (Eq. (8)). $R(V_g, T)$ is taken from Bollinger *et al.*³⁰ in the temperature range from 4.4 K to 100 K. The respective values for $R_0(x)$, $T_c(V_g)$ and \tilde{b}_R are shown in Figs. 2 and 3. The dashed line is the critical BKT behavior given by Eq. (4) with $\tilde{b}_R = b_R/T_c^{1/2} = 4.55$.

at the BKT transition temperature T_c the characteristic BKT form $V \propto I^a$ with $a = 3$.²⁵ Consistency with the characteristic temperature dependence of the sheet resistance (Eq. (4)) was established.^{26,39,40} It was also shown that the effective thickness of the superconducting 2D system can be extracted from the magnetic field dependence of the conductivity at T_c .⁴¹ The gate voltage tuned BKT phase transition line, $T_c(V_g)$, derived from the temperature dependence of the sheet resistance at various gate voltages uncovered with Eq. (4) consistency with $T_c(V_g) = 8.9 \times 10^{-3} (V_g - V_{gc})^{2/3}$ K pointing to quantum critical behavior (Eq. (3)) with $z\bar{\nu} = 2/3$ and the critical sheet resistance $R_c = 2.7 \text{ k}\Omega$.^{26,40} Furthermore the estimates $\bar{\nu} \simeq 2/3$ and $z \simeq 1$, confirming $z\bar{\nu} = 2/3$, have been derived from $R_0(V_g)$.²⁶ This suggests that the gate voltage tuned QSI transition of the 2D electron system at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface belongs to the $(2+z) = 3$ -xy universality class where hyperscaling and with that the proportionality between $d/\lambda^2(0)$ and T_c (Eq. (11)) applies. On the other hand because $\bar{\nu} \simeq 2/3 < 2/D = 1$ disorder is according to Harris theorem relevant as well.^{27,28}

To comment the difficulties in observing the BKT features in the magnetic penetration depth as well as the relation between $d/\lambda^2(0)$ and T_c , we reproduced in Fig. 7 the data of Bert *et al.*³² for a gate voltage tuned superconducting $\text{LaAlO}_3/\text{SrTiO}_3$ interface in terms of T_c vs. $d/\lambda^2(T = 0.04\text{K})$. T_c is defined here as the temperature at which the diamagnetic screening drops below the noise level corresponding to a detectable d/λ^2 of $0.10 - 0.34 \text{ cm}^{-1}$. A glance at Fig. 7 reveals that this is just the regime where the universal quantum behavior applies, indicated by the dashed and solid lines, corresponding to the lower bound (12) and the behavior de-

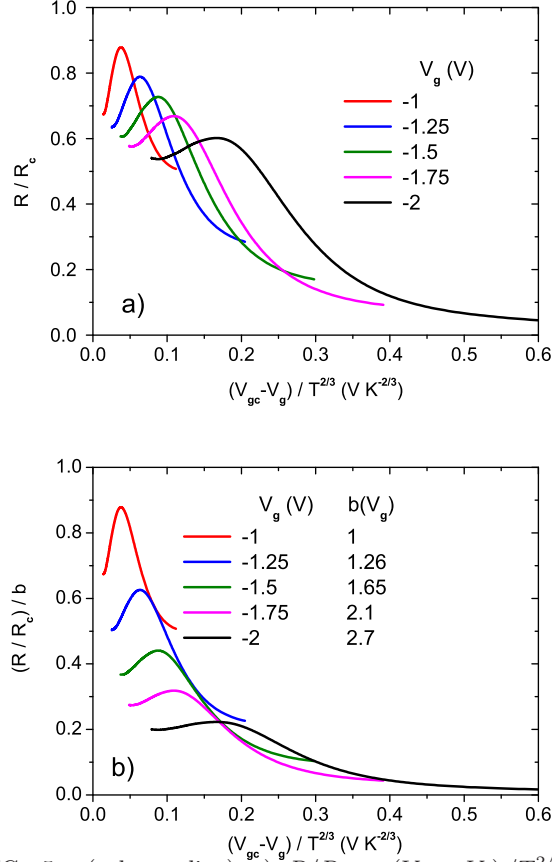


FIG. 5: (color online) a) R/R_c vs. $(V_{gc} - V_g)/T^{2/3}$ with $R_c = 6.45$ k Ω and $V_{gc} = -0.7$ V derived from the temperature dependence of the sheet resistance of Bollinger *et al.*³⁰ The data cover the range from 4 K to 100 K. b) $R/b(V_g)R_c$ vs. $(V_{gc} - V_g)/T^{2/3}$ with $b(V_g = -1V) = 1$ and $b(V_g)$ adjusted to achieve a piecewise collapse of the data.

rived from the ^4He data of Crowell *et al.*,²¹ respectively. Nevertheless the data reveal the flow to the 2D-QSI transition which is attained at much lower T_c 's. Otherwise the data points resemble the outline of a fly's wing,¹⁷ remarkably similar to the T_c vs. $1/\lambda_{ab}^2$ ($T = 0$) plots of the bulk superconductors $\text{Y}_{0.8}\text{Cu}_{0.2-123}$, Tl-1212 ,⁴² and Tl-2201 ,⁴³ covering nearly the doping regime of the so called superconducting dome extending from the underdoped to the overdoped limit. According to the generic plot $T_c/T_c(x_m)$ vs. $\gamma(x_m)/\gamma(T_c)$, where $\gamma = \xi_{ab}/\xi_c$ is the anisotropy and $\xi_{ab,c}$ denote the in-plane and c-axis correlation length, these cuprates become nearly 2D in the underdoped limit.¹⁷ In any case, Fig. 7 shows that in this plot the universal QSI behavior is attained at comparatively low T_c values only. Accordingly, in the QSI regime of interest the Nelson-Kosterlitz jump given by Eq. (10) becomes very small and appears to be beyond present experimental resolution.³² On the contrary in the temperature dependence of the sheet resistance is the BKT critical regime accessible because $T_c/T_{c0} \simeq 0.75$,⁴¹ even though small compared to that in the $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

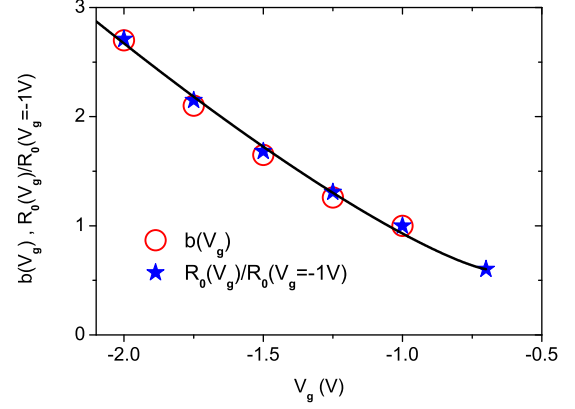


FIG. 6: (color online) $b(V_g)$ and $R_0(V_g)/R_0(V_g = -1V)$. The solid line is $R_0(V_g)/R_0(V_g = -1V) = (R_c + 15.9(V_{gc} - V_g)^{1.26})/R_0(V_g = -1V)$ with $R_c = 6.45$ k Ω , $V_{gc} = -0.7$ and $R_0(V_g = -1V) = 10.7$ k Ω taken from Fig. 2b.

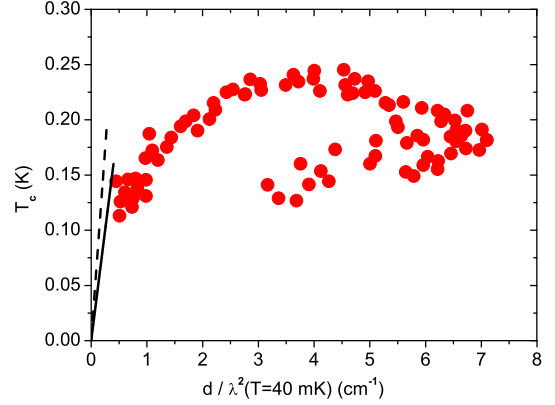


FIG. 7: (color online) T_c vs. d/λ^2 ($T = 0.04$ K) for a gate voltage tuned superconducting $\text{LaAlO}_3/\text{SrTiO}_3$ interface taken from Bert *et al.*³² The solid and dashed lines are the predicted universal relationship (9) close to the QSI transition with $Q_2 = 2/\pi$, resulting from the lower bound (12), and $Q_2 = 1.12$ derived from the ^4He film data of Crowell *et al.*²¹

films where $T_c/T_{c0} \approx 0.1$.

IV. SUMMARY AND DISCUSSION

In sum, we sketched and explored the implications of Berezinskii-Kosterlitz-Thouless (BKT) critical behavior on the quantum critical properties of a two dimensional (2D) quantum superconductor-insulator transition (QSI) driven by the tuning parameter x . It was shown that the finite temperature BKT scenario, implies in terms of the characteristic temperature dependence of the BKT correlation length an explicit quantum scaling function for the sheet resistance $R(x, T)$ along the su-

perconducting branch ending at the nonuniversal critical value $R_c = R_0(x_c)$. This scaling form fixes the BKT-transition line $T_c(x)$ and provides estimates for the quantum critical exponent product $z\bar{\nu}$. In addition, independent estimates of $z\bar{\nu}$, z and $\bar{\nu}$ follow from the x dependence of the nonuniversal parameters $T_c(x)$ and $R_0(x)$ entering the characteristic BKT expression for the sheet resistance $R(x, T)$. This requires that the BKT critical regime where phase fluctuations dominate is attained and the finite temperature BKT relation for the sheet resistance applies for any $T \geq T_c(x) > 0$. The last condition is satisfied because the BKT expression for the sheet resistance is simply related to the characteristic temperature dependence of the BKT correlation length. Quantum fluctuations enter via the nonuniversal parameters $T_c(x)$ and $R_0(x)$ disclosing close to x_c the respective quantum critical behavior. Even though BKT critical behavior is not affected by short-range correlated and uncorrelated disorder^{27,28} the observation of this requires sufficiently homogeneous films and a tuning parameter which does not affect the disorder. Noting that in the magnetic field tuned case there is no BKT-line, thickness and gate voltage tuned 2D-QSI transitions appear to be promising candidates. In any case the scenario outlined here requires a line of BKT-transitions $T_c(x)$ with a quantum critical endpoint $T_c(x_c) = 0$ and sheet resistance data which attain the BKT critical regime.

To illustrate the potential and the implications of this scenario we analyzed data of Bollinger *et al.*³⁰ taken on gate voltage tuned epitaxial films of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ that are one unit cell thick. Evidence for dominant phase fluctuations and BKT-critical behavior was established in terms of the temperature dependence of the sheet resistance revealing a large critical regime extending substantially above the lowest attained temperature $T = 4$ K. From the nonuniversal parameters $T_c(x)$ and $R_0(x)$ disclosing the respective quantum critical properties we derived for the quantum critical exponents the estimates: $z\bar{\nu} \simeq 1.46$ from $T_c(x)$, $\bar{\nu} \simeq 0.63$ and $z \simeq 2.35$ from $R_0(x)$, yielding $z\bar{\nu} \simeq 1.48$. Thus, in contrast to the standard quantum scaling approach, providing an estimate for $z\bar{\nu}$ only, the BKT scenario uncovered $z\bar{\nu}$, $\bar{\nu}$ and z from the quantum critical behavior disclosed in $T_c(x)$ and $R_0(x)$. Additional evidence for $z\bar{\nu} \simeq 1.5$ was established from the comparison of the scaled data with the explicit scaling BKT scaling form of the superconductor branch. We observed that the scaled data does not fall entirely on the BKT curve. Instead a flow to and away from the universal characteristics occurred. As T_c/T decreases for fixed T_c a crossover to AL-behavior sets in, while the rounding of the transition leads with increasing T_c/T to a flow away from criticality. The important lesson then is that the quality of the data collapse on a single curve heavily depends on the temperature range of the data entering the plot. Another striking feature is the extended scaling regime. Within the BKT scenario it follows from the fact that the explicit scaling form of the sheet resistance applies along the entire BKT transition

line irrespective of the distance from the QSI transition. In the quantum scaling approach this property remains hidden and merely suggests an extended quantum critical regime. The piecewise excellent quality of the data collapse also reveals that the provision of the substantial variation of $R_0(x)$ is essential, while in the quantum scaling approach it is fixed by the critical sheet resistance. Supposing that the equivalence between quantum phase transitions with D spatial dimensions and the ones of classical phase transitions in $(D+z)$ dimensions applies, the 2D-QSI transition at the endpoint of a BKT line $T_c(x)$ should belong to the finite temperature $(2+z)$ - xy universality class. Our estimate $z \simeq 2.35$ and with that $D = 4.35 - xy$ critical behavior where $\bar{\nu} = 1/2$. However the fate of this clean critical point under the influence of disorder is controlled by the Harris criterion.^{27,28} If the inequality $\bar{\nu} \geq 2/D$ is fulfilled, the disorder does not affect the critical behavior. If the Harris criterion is violated ($\bar{\nu} < 2/D$), the generic result is a new critical point with conventional power law scaling but new exponents which fulfill $\bar{\nu} < 2/D$. Since $\bar{\nu} = 1/2$ violates this inequality disorder is relevant and drives the system from the mean-field to an other critical point with different critical exponents as our estimate, $\bar{\nu} \simeq 0.63$, consistent with $\bar{\nu} < 2/D = 1$, uncovers. The resulting 2D-QSI transition with $\bar{\nu} \simeq 0.63$ and $z \simeq 2.35$ violates then the equivalence between quantum phase transitions in systems with D spatial dimensions and the ones of classical phase transitions in $(D+z)$ dimensions. In addition the proportionality between $d/\lambda^2(0)$ and T_c (Eq. (11)), valid below the upper critical dimension $D_u = 4$, does no longer hold because $(D+z) \simeq 4.35$ is above $D_u = 4$. In fact magnetic penetration depth measurements taken on underdoped high quality $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ single crystals revealed $T_c \propto (d/\lambda^2(0))^{0.61}$.³⁸ Another option is that the disorder destroys the QSI transition. Given the evidence for a rounded BKT transition (see Fig. 1) and noting that the analyzed data do not extend very close to the QSI transition (see Fig. 4) further studies are required to elucidate this option. In any case, our estimate $\bar{\nu} \simeq 0.63$, the evidence for BKT behavior at finite temperature and the Harris theorem imply the presence and relevance of disorder at the QSI transition and its irrelevance at finite temperature. Further evidence for the importance of disorder follows from the temperature dependence of the sheet resistance in the insulating phase. Considering $R(Vg = 0, T)$ of Bollinger *et al.*³⁰ we observe consistency with the Mott variable range hopping model in 2D. The conductivity exhibits the characteristic temperature dependence $\sigma = \sigma_0 \exp\left(-(T_0/T)^{1/(D+1)}\right)$ which applies to strongly disordered systems with localized states.⁴⁴

A previous and analogous analysis of the sheet resistance data of the superconducting $\text{LaAlO}_3/\text{SrTiO}_3$ interface revealed the critical sheet resistance $R_c = 2.7 \text{ k}\Omega$ and the exponents $z\bar{\nu} \simeq 2/3$, $\bar{\nu} \simeq 2/3$ and $z \simeq 1$.²⁶ In this case the gate voltage tuned QSI transition of the 2D electron system at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface has a finite

temperature counterpart in $(D + z)$ dimensions namely the $(2 + z) = 3$ -xy model where hyperscaling and with that the proportionality between $d/\lambda^2(0)$ and T_c (Eq. (11)) applies. On the other hand $\bar{\nu} \simeq 2/3 < 2/D = 1$ implies according to Harris theorem^{27,28} that disorder is relevant as well.

To comment on the BKT-features in the temperature dependence of the magnetic penetration depth we considered the data of Bert *et al.*³² for a gate voltage tuned superconducting $\text{LaAlO}_3/\text{SrTiO}_3$ interface in terms of T_c vs. $d/\lambda^2(T = 0.04 \text{ K})$. The data reveals the flow to the universal relationship (11) but much lower T_c must be at-

tained to reach quantum critical regime. However in this low T_c regime is the Nelson-Kosterlitz jump given by Eq. (10) very small and appears to be beyond present experimental resolution.³² Otherwise the data points resemble the outline of a fly's wing,¹⁷ remarkably similar to the T_c vs. $1/\lambda_{ab}^2(T = 0)$ plots of the bulk superconductors $\text{Y}_{0.8}\text{Cu}_{0.2}\text{-123}$, Tl-1212 ,⁴² and Tl-2201 ⁴³, covering nearly the entire doping regime in the so called superconducting dome extending from the underdoped to the overdoped limit.

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